

# PSO Based Model Reference Adaptive PI Controller for a Conical Tank Level Process

R. Raghuraman<sup>1\*</sup> and M. Senthilkumar<sup>2</sup>

<sup>1&2</sup>Department of Electronics and Instrumentation Engineering, Annamalai University, Tamil Nadu, India

\*Corresponding author

E-Mail: raghuraman1982@rediffmail.com

**Abstract** - Conical tanks are mostly used in various process industries, such as metallurgical industries, food processing industries, concrete mixing industries wastewater treatment industries etc. A conical tank is basically a nonlinear process as its area of cross section varies with respect to level. This paper describes the implementation of PSO based Model Reference Adaptive PI controller for a nonlinear Conical Tank Level System (CTLs). The mathematical model of CTLs is developed and PSO based Model Reference Adaptive (MRA) PI Controller is proposed for this level system. A result of proposed controller is compared with GA based MRA-PI, MRA-PI and conventional PI controllers to analyze the performance in terms of integral square error and Integral absolute error. The results proved that the superiority of proposed controller.

**Keywords:** Conical Tank, PI Controller, MRAC, PSO

## I. INTRODUCTION

In most of the process industries, control of chemical process system is challenging problems due to their nonlinear dynamic behavior. In particular, one of the nonlinear systems like conical tank is extensively used in process industries, petrochemical industries, food process industries and wastewater treatment industries. Control of such conical tank is a difficult problem because of its non-linearity and changing in area of cross section. Conventional Controllers are normally used in process industries as they are simple and familiar to the field operator but it gives poor performance because of its improper tuning. The variations in the process parameters can be trowned by persistent tuning of the controller parameters using adaptive intelligent techniques like adaptive based control strategy.

One of the most frequently used adaptive control technique is Model Reference Adaptive Control systems (MRAC). This control system [1]-[4] has received considerable attention, and many new approaches have been applied to practical process. Ayachi errachdi *et al.*, [5] presented a Direct Model Reference Adaptive Control (MRAC) for nonlinear time-varying system. The control strategy is based on two-steps; the first is initialization parameters of the controller using reduced number of observation.

In this paper the Conical tank level system has been considered as a typical representative of inherently nonlinear system, thus it is an ideal choice for testing the

modeling capability of the PSO based MRA-PI controller. The main contributions of this paper are the performance of the PSO based MRA-PI control Strategy on the model of the conical tank level system through simulation studies. In section 2 the process description of conical tank is summarized. The MRAC is discussed in section 3. The design and structure of PSO based MRA-PI control strategy is detailed in section 4. Simulation results are analyzed in section 5. Finally, section 6 is summing up of the entire work.

## II. PROCESS DESCRIPTION AND MATHEMATICAL MODELING

Fig.1 shows the Schematic of Conical tank level system. Here  $F_i$  is the inlet flow rate to the tank,  $F_o$  be the outlet flow rate from the tank,  $F_L$  be the disturbance applied to the tank.

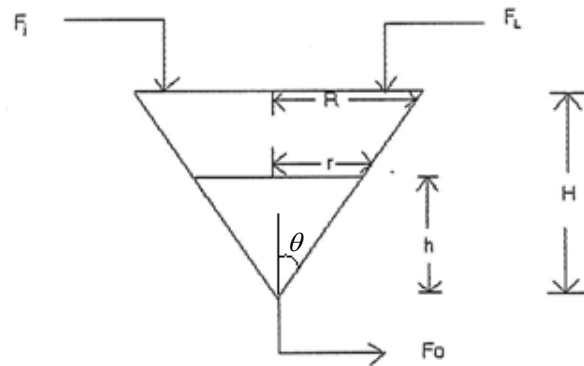


Fig.1 Schematic of Conical Tank Level System

- $F_i$  - Inlet flow rate to the tank ( $m^3 / min$ )
- $F_o$  - Outlet flow rate from the tank ( $m^3 / min$ )
- $F_L$  - Load applied to the tank ( $m^3 / min$ )
- H - Height of the conical tank (m)
- h - Height of the level in the tank at anytime 't' (m)
- R- Top radius of the conical tank (m)
- r- Radius of the conical vessel at a particular level of height h(m)
- A-Area of the conical tank ( $m^2$ )

The nominal operating level h is given by

$$F_{in} - F_{out} = A(h) \frac{dh}{dt} \quad (1)$$

$$\tan\theta = \frac{R}{H} \tag{2}$$

At any level (h)

$$\tan\theta = \frac{r}{h} \tag{3}$$

Equating (2) and (3)

$$\frac{R}{H} = \frac{r}{h}$$

$$r = \frac{Rh}{H} \tag{4}$$

Cross sectional area of the tank at any level (h) is

$$A(h) = \pi r^2 \tag{5}$$

Substitute (4) in (5)

$$A(h) = \frac{\pi R^2 h^2}{H^2} \tag{6}$$

Also

$$F_{out} = b\sqrt{h} \tag{7}$$

Substituting (7) in (1)

$$F_{in} - b\sqrt{h} = A(h) \frac{dh}{dt} \tag{8}$$

$$\frac{dh}{dt} = \frac{F_{in} - b\sqrt{h}}{\pi R^2 h^2 / H^2} \tag{9}$$

From equation (8)

$$F_{in} - \frac{U h}{2h} = A(h) \frac{dh}{dt} \tag{10}$$

Where

$U = b\sqrt{h}$  = Nominal value of outflow rate

Where h and U are nominal values of Process Variable (PV) and Manipulated Variable (MV)

Cross sectional area of conical tank level process is not constant. When the level of the tank varies, cross sectional area of the conical tank also varies. Therefore process parameters are obtained by applying positive and negative step changes in the input. To determine the parameters of the conical tank level process, process reaction curve method is used. The entire operating region is divided into three regions. In the first region the level of the conical tank level process is brought to steady state condition of 10 cm. Then an increase as well as decrease in inflow rate as of equal magnitude is applied. The change in level of the conical tank level process is recorded with respect to time for both cases. The responses is an ‘S’ shaped curve which is known as process reaction curve. From this curve process parameters like process gain ( $K_p$ ), time constant ( $\tau$ ) and dead time ( $t_d$ ) are estimated for both cases and average value is considered.

**A. Determination of Process and Controller Parameters for Conical Tank Level Process:** The process parameters of conical tank level process are obtained by applying step change in the inflow rate. The obtained value of process

parameters are presented in Table I. In this work, PI controller parameters are designed based on Z-N tuning method. The PI mode of control is described by the relationship

$$u(t) = K_c e(t) + \frac{K_c}{T_i} \int_0^t e(t) dt + \text{bias}$$

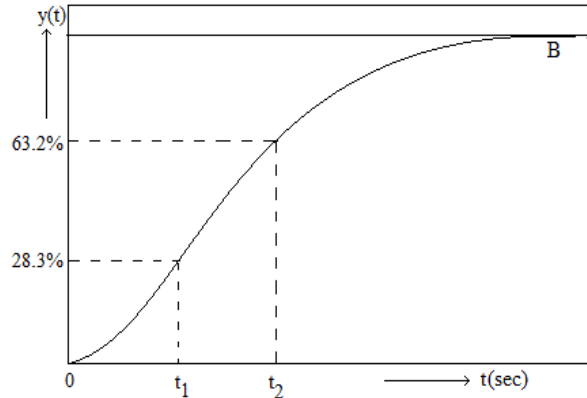


Fig. 2 Process reaction curve

The process parameters are obtained using two point method

$$\text{Time constant } \tau = 1.5(t_2 - t_1) \tag{11}$$

$$\text{Dead time } t_d = t_2 - \tau \tag{12}$$

$$\text{Process gain } K_p = \frac{\Delta \text{Output}}{\Delta \text{Input}} = \frac{B}{M} \tag{13}$$

TABLE I PROCESS PARAMETERS OF CONICAL TANK LEVEL PROCESS FOR VARIOUS REGIONS

Region No.	Range (cm)	Operating point	$K_p$	$\tau$ (min)	$t_d$
1	(0-20)	10	12.64	277.9	7.28
2	(20-40)	30	21.83	4412	68.125
3	(40-60)	50	28.26	14031	133.75

Time instant  $t_2$  is obtained from 63.2% of the final steady state value. Similarly  $t_1$  is obtained from 28.3% of the final steady state value ( $B_u$ ). The PID controller parameters can be determined using Zeigler – Nichols (Z-N) tuning method. The controller parameters are obtained using Zeigler-Nichols open loop tuning method. The obtained controller parameters are presented in Table II.

TABLE II CONTROLLER PARAMETERS OF CONICAL TANK LEVEL PROCESS FOR VARIOUS REGIONS

Region	Nominal operating point	$K_c$	$K_i$
Region 1 (0-20 cm)	10	2.718	24.24
Region2 (20-40 cm)	30	2.67	226.85
Region3 (40-60cm)	50	3.3409	445.38

### III. MODEL REFERENCE ADAPTIVE PI CONTROL

When the plant parameters and the disturbance are varying slowly or slower than the dynamic behavior of the plant, then a MRAC control can be used. The adjustment mechanism uses the adjustable parameter known as control parameter  $\theta$  to adjust the controller parameters. The tracking error and the adaptation law for the controller parameters were determined by MIT rule. MIT (Massachusetts Institute of Technology) Rule is that the time rate of change of  $\theta$  is proportional to negative gradient of the cost function (J) that is:

$$\frac{d\theta}{dt} = -\gamma \frac{\partial J}{\partial \theta} = -\gamma \epsilon \frac{\partial \epsilon}{\partial \theta} \quad (14)$$

The adaptation error  $\epsilon = y_p(t) - y_M(t)$ . The components of  $\frac{d\epsilon}{d\theta}$  are the sensitivity derivatives of the error with respect to the adjustable parameter vector  $\theta$ . The parameter  $\gamma$  is known as the adaptation gain. The MIT rule is a gradient scheme that aims to minimize the squared model error  $\epsilon^2$  from cost function.

$$J(\theta) = \frac{1}{2} \epsilon^2(t) \quad (15)$$

Model reference Adaptive Control: The goal of this section is to develop parameter adaptation laws for a PI control algorithm using MIT rule.

The reference model for the MRAC generates the desired trajectory  $y_M$ , which the conical tank level  $y_p$  has to follow. Standard second order differential equation was chosen as the reference model given by

$$H_M(s) = \frac{b_M}{s^2 + a_{M1}s + a_{M0}} \quad (16)$$

Then the approximate parameter adaptation laws are as follows

$$K_p^* = \left( \frac{-\gamma_p}{s} \right) \epsilon \left( \frac{s}{a_0 s^2 + a_{M1}s + a_{M2}} \right) e \quad (17)$$

$$K_i^* = \left( \frac{-\gamma_i}{s} \right) \epsilon \left( \frac{1}{a_0 s^2 + a_{M1}s + a_{M2}} \right) e \quad (18)$$

Above equations show the change in PID controller parameters with respect to time. By assuming the reference model has 5% maximum overshoot, settling time of 30 seconds and rise time of about 1 second, the second order transfer function of the Model Reference as follows

$$G(s) = \frac{0.3831}{s^2 + 0.2666s + 0.3831}$$

where  $a_0 = 1$ ,  $a_{m1} = 0.2666$  and  $a_{m2} = 0.3831$

### IV. PSO BASED MODEL REFERENCE ADAPTIVE PI CONTROL

In Model Reference Adaptive PI controller  $\gamma_p$  and  $\gamma_i$  values are set by trial and error method. Whereas in PSO based MRA-PI controller  $\gamma_p$  and  $\gamma_i$  values are obtained using PSO.

#### A. Particle Swarm Optimization Algorithm

Particle swarm optimization (PSO) is a population based stochastic optimization technique developed by Dr. Eberhart and Dr. Kennedy in 1995 [6], inspired by social behaviour of bird flocking or fish schooling. Every particle monitors its directions in the issue space which are related with the best arrangement (fitness) it has accomplished up until now. (The fitness value is also stored.) This value is called Personal Best (P Best). Another “best” value that is followed by the particle swarm enhancer is the best esteem, acquired so far by any particle in the neighbours of the particle. This area is called Local best (L best) at the point when a particle takes all the populace as its topological neighbours, the best value is a Global Best and is called G Best. The projected position of  $i^{th}$  particle of the swarm  $x_i$ , and the velocity of this particle  $v_i$  at  $(t+1)^{th}$  iteration are defined and updated as the following two equations,

$$v_i^{t+1} = v_i^t + c_1 r_1 (P_i^t - x_i^t) + c_2 r_2 (g^t - x_i^t) \quad (19)$$

$$x_i^{t+1} = x_i^t + v_i^{t+1} \quad (20)$$

where  $i=1, \dots, n$  and  $n$  is the size of the swarm,  $c_1$  and  $c_2$  are positive constants,  $r_1$  and  $r_2$  are random numbers which are uniformly distributed, determines the iteration number,  $p_i$  represents the best previous position (the position giving the best fitness value) of the  $i^{th}$  particle, and  $g$  represents the best particle among all the particles in the swarm. At the end of the iterations, the best position of the swarm will be the solution of the problem. It cannot be always possible to get an optimum result of the problem, but the obtained solution will be an optimal one [7]-[10].

#### B. Parameters Used For PSO Algorithm

The following parameters are chosen to obtain minimum time domain criteria as a objective function.

The observation time  $T_{ob}=25$  Sec

The step size of the simulation  $H_s=0.001$  Sec,

The average generations =130

The number of particles = 15

The range of  $\lambda$  and  $\delta = 0$  to 2.

The PSO setting parameters are  $c_1=c_2=1.5$ .

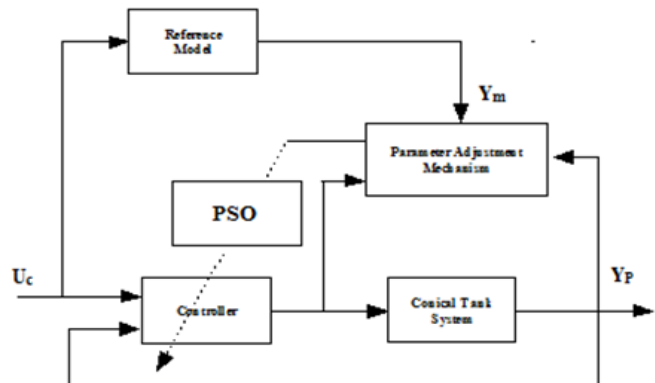


Fig. 3 PSO based MRA-PI control of CTLs

### V. RESULTS AND DISCUSSION

Performances of proposed controller are analyzed using step input at various levels in the CTLS. Initially the tank is maintained at 35 cm of operating level, after that, a step size of 5 cm of level is applied to control loop with PSO based MRA-PI control strategy. In the same way, test runs of GA based MRA-PI, MRA-PI and conventional PI control values are carried out and their responses are presented in Fig.4. It is found that in PSO based MRA-PI makes the system to settle with minimum integral square error at all. To validate the proposed controller, the same procedure is repeated for set point change from 45 to 50 cm and 55 to 60 cm of level are presented in fig.5 and 6 respectively. From the results, the performances are analyzed in terms of ISE and IAE are tabulated in Table III. From Fig 5 and 6, it is also clear that proposed controller tracks the set point quickly with minimum overshoot. The results prove that PSO based MRA-PI controller is appropriate for nonlinear process as it has least error values than the other controller strategies.

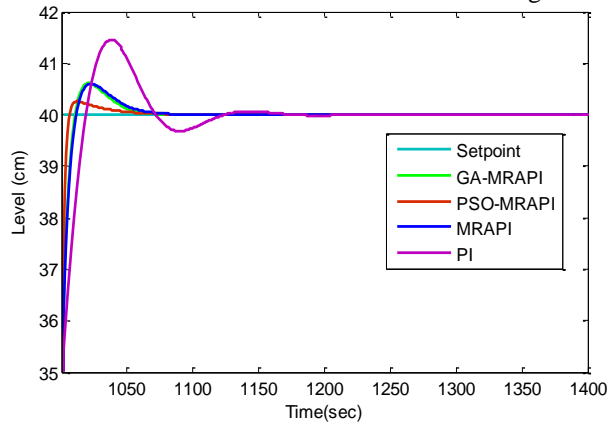


Fig. 4 Servo response of CTLS with PI, MRA-PI, GA-MRAPI and PSO MRA-PI at the operating point of 40

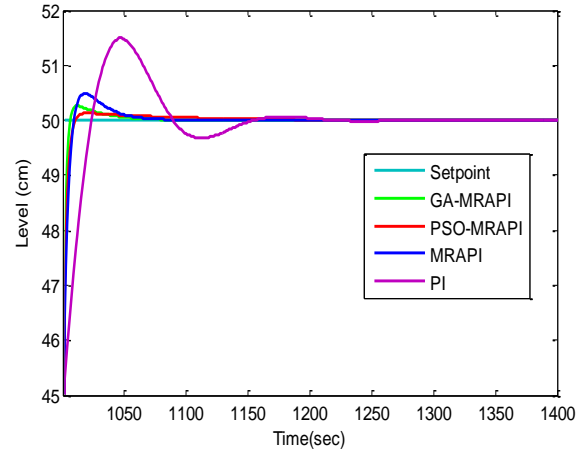


Fig. 5 Servo response of CTLS with PI, MRA-PI, GA-MRAPI and PSO MRA-PI at the operating point of 50

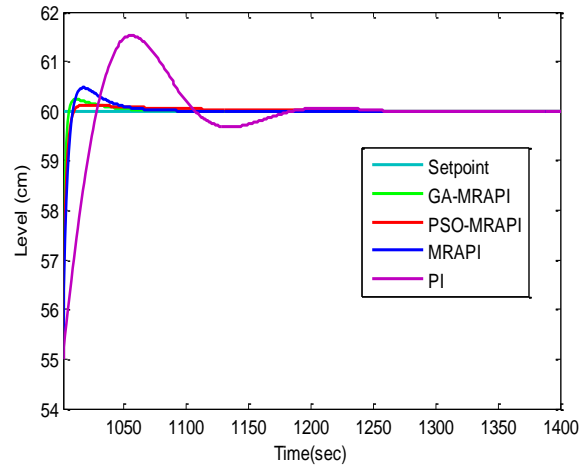


Fig. 6 Servo response of CTLS with PI, MRA-PI, GA-MRAPI and PSO MRA-PI at the operating point of 60

TABLE III PERFORMANCE INDICES AT DIFFERENT OPERATING RANGE

Controller	ISE			IAE		
	OP (35 to 40 cm)	OP (45 to 50 cm)	OP (55 to 60 cm)	OP (35 to 40 cm)	OP (45 to 50 cm)	OP (55 to 60 cm)
PI	145.3	159.8	196.5	64.8	75.3	89.7
MRA-PI	58	40.4	44.6	35.4	27.7	26.2
GA based MRA-PI	55	31.7	31.1	40.16	23.2	21.4
PSO based MRA-PI	21	30.2	22.9	15.1	22.7	14.3

### VI. CONCLUSION

In this paper, PSO based MRA-PI control strategy is developed and implemented for a conical tank level system. This method is suitable for process control applications with a large delay, where a conventional PI controller yield a poor performance. The simulation results are furnished to illustrate the effectiveness of proposed controller with those of GA based MRA-PI, MRA-PI and conventional PI control approaches. The performance indices are also proved that the proposed controller gives a superior performance than the existing control strategies.

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