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An Assessment of Objectivity Convergence of Fuzzy TOPSIS Method Extended With Rank Order Weights in Group Decision Making

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Abstract **- Group decision making in a multi criteria environment is a familiar business situation where the decision makers identify an ideal choice, among many. The situation gets complex when decision makers do not have crisp data to deal with. The fuzzy TOPSIS method, and its likes, provides solution to such problems and the criteria weight plays a determinant role in the overall priority estimation. This paper presents an extended fuzzy TOPSIS approach by incorporating criteria weights derived from rank order. It considers three criteria weights; the rank order centroid, rank sum and rank reciprocal weights. The criteria weights are calculated separately and integrated with fuzzy TOPSIS method to rank choices. Finally, objectivity convergence of the alternative rankings is tested. The proposed method yields a fairly uniform and consistent result in the case of supply chain management and anticipates wide application in multi criteria environment, concomitant with uncertainty and vagueness.**
Keywords: **MCDM, Fuzzy TOPSIS, Rank Order Centroid, Rank Sum, Rank Reciprocal**

I. INTRODUCTION

Decision making is an integral component of every business. Many theorist and practitioners have emphasized its importance as a core managerial function. Its relevance can be envisaged since no managerial function can be accomplished without it. The intrinsic dynamic nature with outcome generating capabilities makes decision making play a pivotal role in the success of an organization. Dale (1960) classifies business decisions as policy, administrative and executive decisions. The former decision class comprises the more complex, strategic ones and is a prerogative of the top management. Administrative decisions are tactical in nature and are primarily within the jurisdiction of the middle level managers. The executive decisions form the regular and routine ones. Irrespective of the nature and level at which decisions are taken, managers are wary of the fact that incorrect and poor decisions have damaging effects on their business in the form of decaying productivity, profitability, competitive edge, culture and may even lead to loss of opportunity identification, thus making it extremely critical and delicate. Furthermore, decision making has evolved from individual to group level owing to the fact that a collective and collaborative decision allows variety in views, more information and better insight. Since, group decision making involves individuals, their decisions cannot be claimed to be devoid of subjectivity. Such nature of it has engaged researchers and practitioners in developing methodologies that supports decision making with higher precision and consistency. Group decision making methods have matured over the years, more so with the advancements in computing abilities and development of sophisticated software. With reference to supply chain management, selection of suppliers has emerged extremely critical owing to its complex nature and the ability to impact business performance severely. Evaluation, ranking and selection of suppliers, also called alternatives, are in most cases done on the basis of multiple and conflicting criteria and thus considered as a multi criteria decision making (MCDM) problem (Shemshadi, Shirazi, Toreihi & Tarokh, 2011).

A wide range of mathematical methods have been developed to address the MCDM problems with realistic and precise solutions (Boer, Labro, & Morlacchi, 2001). The various MCDM methods for supplier selection include technique for order preference by similarity to ideal solution (TOPSIS), analytic hierarchy process (AHP), analytic network process (ANP), data envelopment analysis (DEA), genetic algorithm (GA), goal programming (GP), simple multi-attribute rating technique (SMART), along with some other methods (Dahel, 2003). Since most decisions in the present day business environment are based on group's collective opinion, such judgement is always associated with elements of vagueness, uncertainty and ambiguity. Wang and Lee (2007) classified MCDM problems in two categories, the classical MCDM problems where crisp numbers represent both alternative performance and criteria importance (Feng & Wang, 2000) and the fuzzy MCDM problems where both alternative evaluation and criteria rating are based on vague and uncertain data (Wang, Lee, & Lin, 2003). One may find enormous application of fuzzy MCDM methods in supplier selection. Ding (2011) integrated fuzzy TOPSIS with fuzzy weights. Ghodsypour and O'Brin (1998) framed a decision support system by integrating fuzzy TOPSIS with AHP and linear programming. Liu and Hai, (2005) also extended fuzzy TOPSIS with AHP. Cheong, Jie, Meng & Lan (2008) used fuzzy AHP to develop a decision making system. Chan and Kumar (2007) analyzed global supplier development using fuzzy extended ANP. Wang and Lee (2007) developed a Fuzzy TOPSIS approach based on subjective and objective weights. DEA has been used for supplier selection (Mehralian, Gatari, Morakabati & Vatanpour, *et al.*, 2012) and in evaluating vendor performance (Weber, 1996). Sarkis and Talluri (2002) has developed a model based on

SMART. S.Kumar, S.Kumar and Barman (2018) extended fuzzy TOPSIS with multi criteria goal programming (MCGP). Researchers across the globe have evolved with several modified TOPSIS methods in a group decision making environment (Saghafian & Hejazi, 2005). In most of the group decision making approaches the relative importance of criteria involving subjective assessment have primarily been obtained from the AHP weight and fuzzy weight determination approaches. This paper proposes to introduce an approach to prioritize suppliers that is an extension of fuzzy TOPSIS method by integrating it with subjective weights of criteria based on rank order weighting methods. It also aims to apply it in a real life case and evaluate consistency in results among the varying criteria weight options. The simplicity in assessing rank order weights which also promises reasonably good output consistency actuated consideration of it in this study. The proposed approach uses three different rank order weight measurements that include rank order centroid, rank sum and rank reciprocal methods. The suppler priority is separately evaluated with three different subjective weights and consistency of fuzzy TOPSIS output i.e. supplier ranks checked. The proposed method, applied on a real life case, anticipates enrichment of business decision making in a fuzzy multi criteria decision making environment. The remainder of this paper is structured as follows: Section 2 details the theoretical framework of fuzzy TOPSIS. Section 3 reviews the basics of fuzzy set theory and elaborates on extension of fuzzy TOPSIS method. Section 4 highlights the research framework and defines the fuzzy MCDM problem. Section 5 presents the problem solving approach and findings while section 6 concludes.

II. THEORETICAL FRAMEWORK OF FUZZY TOPSIS

Technique for Order Preference by Similarity to Ideal Solution (TOPSIS) is one of the known classical and popular methods of multiple criteria decision making. The TOPSIS method, first proposed by Hwang and Yoon (1981), is based on the fundamental principle that the best alternative should have the shortest distance from the positive ideal solution and farthest distance from the negative ideal solution (Wang & Elhag, 2006). Here, the ideal solution (also called the positive ideal solution), maximizes the benefit criteria and minimizes the cost criteria, while the negative ideal solution (also called the anti-ideal solution) maximizes the cost criteria and minimizes the benefit criteria (Ghazanfari & Jafari, 2014). The classical TOPSIS approach uses performance of alternatives with respect to each criteria and considers precise value of criteria weights (Dyer, 1992). However, in real world, it is often not possible to get crisp data of alternative's performance with respect to individual criterion. In such situations, the fuzzy approach finds its worth over the classical approach and solutions may be arrived at by assigning relative importance of criteria and alternatives using fuzzy numbers instead of precise numbers (Sun & Lin, 2009; Nadaban, S.Dzitac & I.Dzitac, 2016). Literature also suggests combination of fuzzy TOPSIS with crisp criteria weights. Fuzzy TOPSIS deals with mathematical operations of fuzzy set theory, we first discuss briefly the basics of fuzzy theory, its definitions and operations, before proposing the fuzzy TOPSIS method extended with rank order weights.

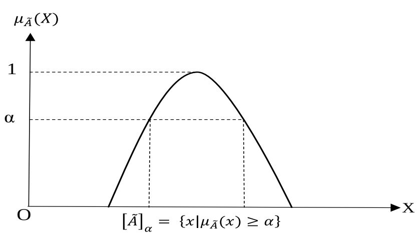
A. Some Basics of Fuzzy Data

The basic concepts, definitions and operations of fuzzy set is presented in the below section.

1. Fuzzy Set

Definition 1: If $\tilde{A}$ is completely characterized by a set of ordered pairs, $\tilde{A}$ is said to represent a fuzzy set (Lai, Liu & Hwang, 1992) and is expressed as $\tilde{A} = \{(x, \mu_{\tilde{A}}(x)) | x \in X, \mu_{\tilde{A}}(x) \in [0,1]\}$. The first element *x* belong to the classical set *A* and the second element, $\mu_{\tilde{A}}(x)$, is the membership function that belongs to the interval [0,1].

Definition 2 (α-cut): The α-cut of a fuzzy set $\tilde{A}$ is a crisp subset of X. It is represented as: $[\tilde{A}]_\alpha = \{x | \mu_{\tilde{A}}(x) \geq \alpha\}$ with $\mu_{\tilde{A}}(x)$ as the membership function of $\tilde{A}$ and $\alpha \in [0,1]$. $[\tilde{A}]_\alpha^L$ and $[\tilde{A}]_\alpha^U$ represents the lower (infimum) and upper points (supremum) respectively of any α-cut, $[\tilde{A}]_\alpha$ (Jahanshahloo, Lotfi & Izadikhah, 2006) and is shown in Figure 1.



Source: Author's nomenclature based on Jahanshahloo *et al.* (2006)

Figure 1 An α-cut example

Definition 3 (Normality): A fuzzy set $\tilde{A}$ is said to be normal iff $sup_x \mu_{\tilde{A}}(x) = 1$ (Jahanshahloo *et al.*, 2006).

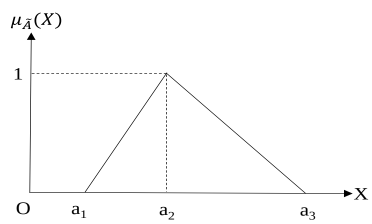
Definition 4 (Convexity): A fuzzy set $\tilde{A}$ in X is convex iff for every pair of point x_1 and x_2 in X, the membership function of $\tilde{A}$ satisfies the inequality $\mu_{\tilde{A}}(\delta x_1 + (1-\delta)x_2 \geq min(\mu_{\tilde{A}}(x_1), \mu_{\tilde{A}}(x_2))$ where $\delta \in [0,1]$ (Jahanshahloo *et al.*, 2006).

Definition 5 (Fuzzy number): A fuzzy number $\tilde{A}$ is said to be a convex normalized fuzzy set $\tilde{A}$ of the real line ℝ with continuous membership function (Jahanshahloo *et al.*, 2006).

Definition 6 (Triangular fuzzy numbers): It is denoted with three points as $\tilde{A} = (a_1, a_2, a_3)$. The left span is represented by a_1, central value by a_2 and the right span by a_2 (Figure 2 – a triangular fuzzy number) and is regarded as a membership function that holds the below mentioned conditions (Gani & Assarudeen, 2012).

1. a_1 to a_2 is increasing function
2. a_2 to a_3 is decreasing function
3. $a_1 \leq a_2 \leq a_3$

$$\mu_{\tilde{A}}(x) = \begin{cases} 0 & for\ x < a_1 \\ \dfrac{x - a_1}{a_2 - a_1} & for\ a_1 \leq x \leq a_2 \\ \dfrac{a_3 - x}{a_3 - a_2} & for\ a_2 \leq x \leq a_3 \\ 0 & for\ x > a_3 \end{cases}$$



Source: Author's nomenclature based on Gani & Assarudeen (2012)

Fig. 2 A triangular fuzzy number $\tilde{A} = (a_1, a_2, a_3)$

Definition 7 (Multiplication of a triangular fuzzy number with a non-fuzzy number): The multiplication operation of a triangular fuzzy number $\tilde{A} = (a_1, a_2, a_3)$ and a non-fuzzy number, α such that $\alpha \in \mathbb{R}_+$ is defined as (Nadaban *et al.*, 2016): $\alpha\ X\ \tilde{A} = (\alpha a_1, \alpha a_2, \alpha a_3)$.

Definition 8: A fuzzy number $\tilde{A}$ is said to be positive if $\mu_{\tilde{A}}(x) = 0$ for all x < 0 (Jahanshahloo *et al.*, 2006).

Definition 9: $\tilde{A}$ is said to be a normalized positive triangular fuzzy number if it satisfies the conditions of $[\tilde{A}]_\alpha^L > 0$ and $[\tilde{A}]_\alpha^U < 1$ for $\alpha \in [0, 1]$ (Jahanshahloo *et al.*, 2006).

Definition 10: (Distance between two triangular fuzzy numbers). Chen (1985) proposed the vertex method of calculating distance between two triangular fuzzy numbers. The distance between two triangular fuzzy numbers $\tilde{A} = (a_1, a_2, a_3)$ and $\tilde{B} = (b_1, b_2, b_3)$ are calculated as:

$$d(\tilde{A}, \tilde{B}) = \sqrt{\frac{1}{3}\left[(a_1 - b_1)^2 + (a_2 - b_2)^2 + (a_3 - b_3)^2\right]}$$

B. Extension of Fuzzy TOPSIS

A systematic approach in developing fuzzy TOPSIS begins with identification of alternatives to be ranked along with the conflicting criteria with respect to which the alternatives or available choices are judged. We denote $A_1, A_2, \ldots, A_m$ as *'m'* identified alternatives and $C_1, C_2, \ldots, C_n$ as '*n'* different criteria.

In the next step, the fuzzy decision matrix is constructed with the performance of alternatives, $\tilde{x}_{ij}$, which is a fuzzy number, and is obtained from the rating of alternative A_i with respect to criterion C_j that is given by a particular decision maker (Jahanshahloo *et al.*, 2006). For any MCDM involving fuzzy data, the fuzzy decision matrix may be represented as Figure 3.

	C_1	C_2	...	C_n
A_1	$\tilde{x}_{11}$	$\tilde{x}_{12}$	...	$\tilde{x}_{13}$
A_2	$\tilde{x}_{21}$	$\tilde{x}_{22}$	...	$\tilde{x}_{2n}$
A_m	$\tilde{x}_{m1}$	$\tilde{x}_{m2}$	...	$\tilde{x}_{mn}$

Source: Author's nomenclature based on Jahanshahloo *et al.* (2006)

Fig. 3 Fuzzy decision matrix

If the total number of decision makers are K, the fuzzy rating for alternative A_i against criterion C_j, given by the k^{th} decision maker is represented as $\tilde{x}_{ij}^k = (a_{ij}^k, b_{ij}^k, c_{ij}^k)$ and the fuzzy weight of criterion C_j given by the k^{th} decision maker is expressed as $\widetilde{w}_j^k = (w_{j1}^k, w_{j2}^k, w_{j3}^k)$ (Nadaban *et al.*, 2016). Both $\tilde{x}_{ij}^k$ and $\widetilde{w}_j^k$ are expressed as fuzzy triangular numbers. In general a fuzzy triangular number representing i^{th} alternative against j^{th} criteria may be expressed as $\tilde{x}_{ij} = (a_{ij}, b_{ij}, c_{ij})$ and the fuzzy weight of j^{th} criteria represented as $\widetilde{w}_j = (w_{j1}, w_{j2}, w_{j3})$. Application of TOPSIS with fuzzy data includes certain steps shown in the subsequent section.

C. Steps Involved in Developing Extended Fuzzy TOPSIS

1. Identifying criteria to be used for evaluating the chosen alternatives.
2. Selecting alternatives those are to be ranked against the identified criteria.
3. Evaluating the values or ratings of each alternative with respect to the criterions. The linguistic values obtained from the judgement of decision makers are converted to triangular fuzzy number using the linguistic variable scale introduced by Zadeh (1970).
4. Identification of criteria weights (w_i).

The criteria weights indicate their relative importance that are critical in the overall decision making. According to Chen, Tzeng and Ding (2003) evaluation of criteria entails diverse opinions and meanings, and thus it is worthwhile not to assume equal importance of each criterion. Literature suggests two approaches to weight determination. The subjective approaches depend on the decision maker's judgement and cognition while the other approach includes objective methods that use mathematical models without any consideration of the decision maker's judgement. It also follows from literature that there are various subjective methods for criteria weight determination but little agreement exist on the most accurate method since calculation of weights depend on the method chosen (Barron & Barrett, 1996a, 1996b). Ranked weights are expected to yield a fairly good estimation of relative importance of criteria or attributes and rank order weights are generated from the below two steps:

a. Ranking by decision makers as per their understanding of criterion importance and

b. Calculating weights for each rank using the appropriate rank order weighting methods

Two functions, rank reciprocal and rank sum (Stillwell, Seaver & Edwards, 1981) were initially developed followed by rank centroid method for the purpose of weight determination method (Solymosi & Dompi, 1985; Barron, 1992). We have used all the three approaches in the study owing to their simplicity and effectiveness. The weight calculation procedures are presented below.

The rank sum (RS) method calculates weights (Stillwell *et al.*, 1981) as:

$$W_j\,(RS) = \frac{n - r_j + 1}{\sum_{k=1}^{n}(n - r_k + 1)} = \frac{2(n + 1 - r_j)}{n(n+1)}$$ where r_j: rank of j^{th} criterion, j=1,2,...,n & r_k: k^{th} rank

The rank reciprocal or inverse (RR) method calculates criteria weights (Stillwell *et al.*, 1981) as:

$$W_j\,(RR) = \frac{\frac{1}{r_j}}{\sum_{k=1}^{n}\left(\frac{1}{r_k}\right)}$$ where r_j: rank of j^{th} criterion, j=1,2,...,n & r_k: k^{th} rank

The rank-order centroid (ROC) method estimates criteria weights that are based on minimizing the maximum error of each weight through centroid identification for all possible weights. Weights obtained by this approach were found to be very stable (Barron & Barrett, 1996a). This method was generalized for n > 2 and with more criteria the error for rank criteria is even lesser (Barron & Barret, 1996). Rank order centroid weight is calculated as:

$$W_j\,(ROC) = \frac{1}{n}\sum_{k=1}^{n}\frac{1}{r_k}$$ where r_k is the k^{th} rank

5. Constructing the fuzzy decision matrix where each $\tilde{x}_{ij}$ is triangular fuzzy number i.e. $\tilde{x}_{ij}^{k} = \left(a_{ij}^{k}, b_{ij}^{k}, c_{ij}^{k}\right)$. Transformation of triangular fuzzy numbers, which is the aggregate fuzzy ratings, to fuzzy decision matrix and it is obtained as (Nadaban *et al.*, 2016):

$$a_{ij} = \min_{k}\{a_{ij}^{k}\}, b_{ij} = \frac{1}{K}\sum_{k=1}^{K} b_{ij}^{k}, c_{ij} = \max_{k}\{c_{ij}^{k}\}$$

6. Constructing the normalized fuzzy decision matrix using the concept of α-cut (Definition 2) as per Jahanshahloo *et al.* (2006). The subsequent section details the steps involved in construction of normalized decision matrix with triangular fuzzy numbers.

a. In the first step, a set of α-cut is calculated and each of the fuzzy numbers are transformed to an interval fuzzy number as $\tilde{x}_{ij} = \left[[\tilde{x}_{ij}]_{\alpha}^{L}, [\tilde{x}_{ij}]_{\alpha}^{U}\right]$ (Jahanshahloo *et al.*, 2006).

b. In this step, the fuzzy interval number is transformed to its normalized interval as:

$$[\tilde{n}_{ij}]_{\alpha}^{L} = [\tilde{x}_{ij}]_{\alpha}^{L} \Big/ \sqrt{\sum_{i=1}^{m}\left([\tilde{x}_{ij}]_{\alpha}^{L}\right)^{2} + \left([\tilde{x}_{ij}]_{\alpha}^{U}\right)^{2}},$$

i = 1, 2, …, m and j = 1, 2,…., n

$$[\tilde{n}_{ij}]_{\alpha}^{U} = [\tilde{x}_{ij}]_{\alpha}^{U} \Big/ \sqrt{\sum_{i=1}^{m}\left([\tilde{x}_{ij}]_{\alpha}^{L}\right)^{2} + \left([\tilde{x}_{ij}]_{\alpha}^{U}\right)^{2}},$$

i = 1, 2, …, m and j = 1, 2,…., n

where $\left[[\tilde{n}_{ij}]_{\alpha}^{L}, [\tilde{n}_{ij}]_{\alpha}^{U}\right]$ is the normalized fuzzy interval of the fuzzy interval $\left[[\tilde{x}_{ij}]_{\alpha}^{L}, [\tilde{x}_{ij}]_{\alpha}^{U}\right]$.

c. In the final step of constructing a fuzzy decision matrix, the normalized interval is transformed into normalized triangular fuzzy number $\tilde{N}_{ij} = (n_{ij}, p_{ij}, q_{ij})$. By setting α = 1, we get the left span of the triangular fuzzy number as $\tilde{n}_{ij} = [\tilde{n}_{ij}]_{\alpha=1}^{L} = [\tilde{n}_{ij}]_{\alpha=1}^{U}$ and by setting α = 0, we get the mid value and the right span. The mid value is obtained from $[\tilde{n}_{ij}]_{\alpha=0}^{L} = n_{ij} - p_{ij}$ and the right span from $[\tilde{n}_{ij}]_{\alpha=0}^{U} = n_{ij} + q_{ij}$. Thus, the mid value $p_{ij} = n_{ij} - [\tilde{n}_{ij}]_{\alpha=0}^{L}$ and the right span $q_{ij} = [\tilde{n}_{ij}]_{\alpha=0}^{U} - n_{ij}$. The normalized fuzzy decision matrix is created from $\tilde{N}_{ij}$.

7. Constructing weighted normalized fuzzy decision matrix $\tilde{v}_{ij}$ from the criteria weights and fuzzy decision matrix as per *Definition 7*. Thus, $\tilde{v}_{ij} = \tilde{N}_{ij}\,.\,\tilde{w}_j$, from each $\tilde{v}_{ij} \in [0,1]$.

8. Identifying fuzzy positive ideal solution (FPIS) and fuzzy negative ideal solution (FNIS) as (Jahanshahloo *et al.*, 2006; Wang & Lee, 2009):

$$A^{+} = \{\tilde{v}_1^{+}, \dots\dots, \tilde{v}_n^{+}\} = \left\{\left(\max_{j} \tilde{v}_{ij} \middle| i \in I\right), \left(\min_{j} \tilde{v}_{ij} \middle| i \in J\right)\right\}$$

$$A^{-} = \{\tilde{v}_1^{-}, \dots\dots, \tilde{v}_n^{-}\} = \left\{\left(\min_{j} \tilde{v}_{ij} \middle| i \in I\right), \left(\max_{j} \tilde{v}_{ij} \middle| i \in J\right)\right\}$$

where *I* and *J* are associated with the benefit and cost criteria respectively.

9. Calculating closeness or separation measure of each alternative from its FPIS (d_i^{+}) and FNIS (d_i^{-}) using vertex method *(Definition 10)*.

10. Finding the closeness coefficient or the relative closeness whose values are used to rank the alternatives. The relative closeness of alternative A_i with respect to A^{+} is defined as (Jahanshahloo *et al.*, 2006): $R_i = d_i^{-} \big/ (d_i^{+} + d_i^{-})$, $R_i \in [0,1]$ *and i = 1, 2, …, m.* Since $d_i^{-} \geq 0$ and $d_i^{+} \geq 0$, $R_i \in [0, 1]$. Alternatives with higher value of R_i have better rank or more preferred compared to those with lower values.

III. RESEARCH FRAMEWORK

This paper is descriptive in nature and relies on cross sectional study design. Primary survey forms the basis of the study. Before conducting the survey, the MCDM problem is first defined from a real life industry situation of supplier selection. A mid-sized adhesive manufacturing company was approached and on explaining the pure research motive behind the study, they introduced to the decision makers in supplier selection. Three members, one each from sales (D1), technical (D2) and marketing (D3) business verticals forms the decision making group. The members participated in criteria identification, alternative selection, evaluated performance of the alternatives against each criteria and assigned ranks to each criteria. The decision making group emerged with three suppliers; Kent (A1), Loctite (A2) and 3M (A3); as alternatives. They also identified six criteria that includes (1) price offered by the suppliers, (2) supplier's capability of delivering products on time, (3) supplier's technical knowhow, (4) supplier's service support level, (5) supplier's track history with the company & (6) supplier's product quality. To arrive at a solution to the decision problem, fuzzy TOPSIS method is chosen due to paucity of crisp data. Alternative prioritization is done on the basis of the extended TOPSIS method as detailed in the previous section. To capture the judgement of the decision makers, a linguistic scale is developed. The scale consists of linguistic terms, each of which correspond to a rating and each rating further correspond to a triangular fuzzy number (TFN) (Table 1). The decision makers rated the criteria and the alternatives with respect to each criteria independently using the linguistic scale.

IV. PROBLEM SOLVING APPROACH AND FINDINGS

The problem solving approach using extended fuzzy TOPSIS method is executed in a step wise manner. The findings of each operation performed are also presented and the overall prioritization of alternatives arrived at. Three sets of ranks are obtained, each with ROC, RS & RR weights.

1. The linguistic judgement of the decision makers is converted to the corresponding fuzzy triangular numbers (Table II).
2. The TFNs that correspond to the performance of suppliers against each criteria is then transformed to fuzzy decision matrix (FDM) (as per *5* and is shown in Table III.
3. The normalized FDM is made from the FDM using the concept of α-cut (as per *6* and is represented in Table IV.

TABLE I LINGUISTIC TERMS FOR ALTERNATIVES RATING

Linguistic Scale	Rating	Triangular Fuzzy Number
Extremely Poor	1	(0.5,0.5,1)
Very Poor	2	(1,1.5,2)
Poor	3	(2,2.5,3)
Medium Poor	4	(3,3.5,4)
Fair	5	(4,4.5,5)
Medium Good	6	(5,5.5,6)
Good	7	(6,6.5,7)
Very Good	8	(7,7.5,8)
Excellent	9	(8,8.5,9)

Source: Author's own scale development based on Zadeh (1970)

TABLE II SUPPLIER RATING WITH RESPECT TO CRITERIA BY DECISION MAKERS (TFN)

Alternatives	Decision Makers	C1	C2	C3	C4	C5	C6
A1	D1	(5,5.5,6)	(1,1.5,2)	(7,7.5,8)	(1,1.5,2)	(7,7.5,8)	(8,8.5,9)
	D2	(5,5.5,6)	(3,3.5,4)	(7,7.5,8)	(0.5,0.5,1)	(7,7.5,8)	(6,6.5,7)
	D3	(4,4.5,5)	(1,1.5,2)	(7,7.5,8)	(1,1.5,2)	(6,6.5,7)	(8,8.5,9)
A2	D1	(8,8.5,9)	(8,8.5,9)	(8,8.5,9)	(6,6.5,7)	(8,8.5,9)	(5,5.5,6)
	D2	(8,8.5,9)	(8,8.5,9)	(7,7.5,8)	(7,7.5,8)	(7,7.5,8)	(6,6.5,7)
	D3	(7,7.5,8)	(8,8.5,9)	(8,8.5,9)	(7,7.5,8)	(8,8.5,9)	(5,5.5,6)
A3	D1	(6,6.5,7)	(4,4.5,5)	(7,7.5,8)	(6,6.5,7)	(7,7.5,8)	(4,4.5,5)
	D2	(7,7.5,8)	(8,8.5,9)	(7,7.5,8)	(5,5.5,6)	(7,7.5,8)	(6,6.5,7)
	D3	(7,7.5,8)	(5,5.5,6)	(8,8.5,9)	(7,7.5,8)	(7,7.5,8)	(3,3.5,4)

Source: Author's calculation

TABLE III FUZZY DECISION MATRIX (FDM)

Alternatives	C1	C2	C3	C4	C5	C6
A1	(4,5.2,6)	(1,2.2,4)	(7,7.5,8)	(0.5,1.2,2)	(6,7.2,8)	(6,7.8,9)
A2	(7,8.2,9)	(8,8.5,9)	(7,8.2,9)	(6,7.2,8)	(7,8.2,9)	(5,5.8,7)
A3	(6,7.2,8)	(4,6.2,9)	(7,7.8,9)	(5,6.5,8)	(7,7.5,8)	(3,4.8,7)

Source: Author's calculation

TABLE IV NORMALIZED FUZZY DECISION MATRIX (NFDM)

Alternatives	C1	C2	C3	C4	C5	C6
A1	(0.13,0.03,0.02)	(0.05,0.03,0.04)	(0.18,0.01,0.01)	(0.03,0.02,0.02)	(0.18,0.03,0.02)	(0.19,0.05,0.03)
A2	(0.21,0.04,0.03)	(0.21,0.01,0.01)	(0.2,0.03,0.02)	(0.18,0.03,0.02)	(0.2,0.03,0.02)	(0.14,0.02,0.03)
A3	(0.18,0.03,0.02)	(0.15,0.05,0.07)	(0.19,0.02,0.03)	(0.16,0.04,0.03)	(0.18,0.01,0.01)	(0.12,0.04,0.05)

Source: Author's calculation

4. The decision makers ranked the criteria individually and the same is shown in Table V. Each of the ranks is converted to ranked weights. The ranked weights of criteria vary depending on the weighting method chosen. Table VI shows varying criteria weights based on ROC, RS and RR weight determination methods.
5. The normalized fuzzy decision matrix (NFDM) thus obtained in step 3 is then transformed to the weighted normalized fuzzy decision matrix (WNFDM) using *7*. It involves multiplication of a triangular fuzzy number with a non-fuzzy number. Since three different rank order weight determination methods are considered in the study, we have three different WNFDMs, one based on ROC, the other based on RS and the third one based on RR method, which are shown in Table VII to IX.

TABLE V RANKING OF CRITERIA BY DECISION MAKERS

Decision Makers	C1	C2	C3	C4	C5	C6
D1	1	3	4	5	6	2
D2	3	2	4	5	6	1
D3	1	3	5	4	6	2

Source: Author's calculation

TABLE VI CRITERIA WEIGHTS BASED ON ROC, RS & RR METHODS

Type of Weights	C1	C2	C3	C4	C5	C6
ROC Weight	0.408	0.242	0.158	0.103	0.061	0.028
RS Weight	0.286	0.238	0.190	0.143	0.095	0.048
RR Weight	0.408	0.204	0.136	0.102	0.082	0.068

Source: Author's calculation

Table VII WEIGHTED NORMALIZED FUZZY DECISION MATRIX BASED ON ROC METHOD

Alternatives	C1	C2	C3	C4	C5	C6
A1	(0.042,0.01,0.007)	(0.009,0.006,0.007)	(0.016,0.001,0.001)	(0.002,0.002,0.002)	(0.005,0.001,0.001)	(0.056,0.015,0.009)
A2	(0.068,0.013,0.01)	(0.039,0.002,0.002)	(0.018,0.003,0.002)	(0.014,0.002,0.002)	(0.006,0.001,0.001)	(0.042,0.006,0.009)
A3	(0.059,0.01,0.007)	(0.028,0.009,0.013)	(0.017,0.002,0.003)	(0.012,0.003,0.003)	(0.005,0,0)	(0.036,0.012,0.015)

Source: Author's calculation

TABLE VIII WEIGHTED NORMALIZED FUZZY DECISION MATRIX BASED ON RS METHOD

Alternatives	C1	C2	C3	C4	C5	C6
A1	(0.033,0.008,0.005)	(0.01,0.006,0.008)	(0.023,0.001,0.001)	(0.003,0.002,0.002)	(0.009,0.001,0.001)	(0.048,0.013,0.008)
A2	(0.053,0.01,0.008)	(0.043,0.002,0.002)	(0.025,0.004,0.003)	(0.02,0.003,0.002)	(0.01,0.001,0.001)	(0.036,0.005,0.008)
A3	(0.046,0.008,0.005)	(0.031,0.01,0.014)	(0.024,0.003,0.004)	(0.018,0.004,0.003)	(0.009,0,0)	(0.03,0.01,0.013)

Source: Author's calculation

TABLE IX WEIGHTED NORMALIZED FUZZY DECISION MATRIX BASED ON RR METHOD

Alternatives	C1	C2	C3	C4	C5	C6
A1	(0.041,0.01,0.006)	(0.008,0.005,0.006)	(0.017,0.001,0.001)	(0.003,0.002,0.002)	(0.012,0.002,0.001)	(0.052,0.014,0.008)
A2	(0.067,0.013,0.01)	(0.033,0.002,0.002)	(0.019,0.003,0.002)	(0.016,0.003,0.002)	(0.014,0.002,0.001)	(0.038,0.005,0.008)
A3	(0.057,0.01,0.006)	(0.024,0.008,0.011)	(0.018,0.002,0.003)	(0.014,0.004,0.003)	(0.012,0.001,0.001)	(0.033,0.011,0.04)

Source: Author's calculation

6. The fuzzy positive ideal solution and fuzzy negative ideal solution are calculated from the weighted normalized fuzzy decision matrix (shown in Tables VII to IX) as per *8*.
7. The three WNFDMs are used to calculate the distance or separation measure of each alternative from its FPIS (d_i^+) & FNIS (d_i^-) as per *9*.
8. Using the separation measures, the relative closeness or closeness coefficient is determined (as per *9*. from which the rank of suppliers is arrived at. Table X shows the FPIS, FNIS, closeness coefficient (relative closeness) and supplier ranking using ROC method of criteria weight determination while Table XI and Table XII shows the same parameters using RS method and RR methods criteria weight assessment respectively.

TABLE X ROC METHOD BASED SEPARATION MEASURES, RELATIVE CLOSENESS & RANK

Alternatives	d_i^+	d_i^-	R_i	Rank
A1	0.0739	0.0675	0.4775	2
A2	0.1185	0.1074	0.4756	3
A3	0.0857	0.0832	0.4925	1

Source: Author's calculation

TABLE XI RS METHOD BASED SEPARATION MEASURES, RELATIVE CLOSENESS & RANK

Alternatives	d_i^+	d_i^-	R_i	Rank
A1	0.0748	0.0654	0.4664	2
A2	0.1228	0.1059	0.4631	3
A3	0.0904	0.0823	0.4765	1

Source: Author's calculation

TABLE XII RR METHOD BASED SEPARATION MEASURES, RELATIVE CLOSENESS & RANK

Alternatives	d_i^+	d_i^-	R_i	Rank
A1	0.0762	0.0759	0.4992	2
A2	0.1170	0.1100	0.4845	3
A3	0.0827	0.1026	0.5536	1

Source: Author's calculation

It is found that the order of prioritization i.e. the rank of suppliers derived with ROC, RS and RR weights are same. This is indeed a good indication to anticipate consistency in results among the three separate measures of weights which remains our prime objective. Thus, the choice of rank order weight appears to have low significance in determining the relative preference in fuzzy decision making environment. Results reveal A3 to be more preferred. Finally, Table XIII summarizes the supplier ranks that have obtained by deploying fuzzy TOPSIS integrated with ROC, RS and RR weights, all of which are based on rank order.

TABLE XIII FUZZY TOPSIS RANKS BASED ON ROC, RS & RR WEIGHTS

Alternatives	Rank *(ROC Wt.)*	Rank *(RS Wt.)*	Rank *(RR Wt.)*
A1	2	2	2
A2	3	3	3
A3	1	1	1

Source: Author's calculation

V. CONCLUSION

Real life business decision making process gets more complex with the non-availability of crisp data and crucial decisions have to be taken based on imprecise i.e. vague data. The domain of fuzzy mathematics offers solution to business practitioners and researchers worldwide in such situations. The present paper has its inspiration rooted in finding an alternative solution to multi criteria business decision problems dealing with imprecise data. The output of the extended fuzzy TOPSIS method is represented by the relative closeness values and ranks derived from it. The value of relative closeness of an alternative is found to be different if one compares three outputs, each based on a different criteria weight determination method. The relative closeness value of supplier A1 is 0.4775, 0.4664 and 0.4992 with ROC, RS and RR methods respectively. Similar behaviour is seen for supplier A2 and A3; however, the order of supplier preference, represented by their ranks have been found to be exactly the same i.e. a very high level of consistency in supplier rank is observed. Thus, in multi criteria group decision making situations where criteria weights have to be determined by subjective methods, rank order weights prove to generate fairly consistent and uniform results. It is also seen that alternative A3 i.e. 3M is the most preferred supplier followed by A1 i.e. Kent and Loctite (A2) in order of decreasing preference. Though fuzzy TOPSIS with different rank order weights have yielded same overall rank of the suppliers, it may be of academic and business interest to find out if other subjective methods yield similar results.

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